R09

Code No: C6105, C6505

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
M.Tech I - Semester Examinations, March/April-2011
DETECTION AND ESTIMATION THEORY
(COMMON TO COMMUNICATION SYSTEMS, WIRELESS AND MOBILE COMMUNICATION)

Time: 3hours **Max. Marks: 60**

Answer any five questions
All questions carry equal marks
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1. Define the Cramer-Rao lower bound (CRLB) for vector parameter estimation. Given the observations x[n] = A + Bn + w[n], n = 0, 1, ... , N - 1 where w[n] is white Gaussian noise. Find the CRLB for the slope *B* and intercept *A*. [12]

2. Suppose *x* is a binary variable that assumes a value of 1 with probability *a* and a value of 0 with probability 1 - *a*. The probability distribution of *x* can be described by $P(x) = (1 - a)^{1-x}a^x$. Suppose we draw a random sample of *N* values $x_1, x_2, \ldots,$ x_N. Find the Maximum Likelihood (ML) estimate of *a*. [12]

3. We observe a random sample *z*(1), *z*(2), ... , *z*(*N*) of an *i.i.d* random variable *z* that is exponentially distributed with parameter θ *i.e.*, $p(z(i)|\theta) = \theta \exp[-z(i)\theta]$ where $\theta \geq 0$ and $z(i) \geq 0$. Also, θ is a random variable with prior probability density function $p(\theta) = \theta \exp(-2\theta)$ where $\theta \geq 0$. Assume that the means and variances of *z*(*i*) and θ are known;
   a) Determine the Maximum A Posteriori estimate, $\hat{\theta}_{MAP}(N)$.
   b) Consider the sample mean of *z*(*i*). As *N* gets large, how does $\hat{\theta}_{MAP}(N)$ compare with the sample mean. [12]

4. Draw the structure of a Kalman filter based state estimation and prediction system clearly indicating the signal flow paths and associated matrices. Also write the associated expressions. [12]

5.a) Derive the decision metric used in Bayes' detector for the binary hypothesis testing problem.
  b) Consider a binary hypothesis testing problem. A signal of 5 volts may or may not be transmitted and is observed under AWGN. The noise samples have zero mean and unit variance. Cost associated with correct decisions are 0 and wrong decisions are 1. Also the *a priori* probabilities are $P_0 = P_1 = 0.5$. If 10 independent samples of the received signal are available, calculate the effective detection threshold, Probability of detection (P_D), Probability of Miss (P_M), and Probability of False Alarm (P_{FA}). [12]

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6.a) Draw typical receiver operating characteristic curves and discuss the inferences which can be obtained from it.

b) Under no signal condition only noise is received, while under the signal present condition, 1 volt plus additive noise is received. Let 50 samples are available at the receiver. The noise samples are Gaussian with zero mean and unit variance. Design a Neyman-Pearson detector for a constant false alarm probability of 0.1. [12]

7. Consider a multiple hypothesis testing problem with three constant signals m_0, m_1 and m_2. The signals are embedded in *i.i.d* Gaussian noise ~ ℵ (0, σ^2) and N samples are available.

$$H_0 = y_n \sim \aleph(m_0, \sigma^2)$$
$$H_1 = y_n \sim \aleph(m_1, \sigma^2)$$
$$H_1 = y_n \sim \aleph(m_2, \sigma^2)$$

Where $n = 0, 1, \ldots\ldots, N - 1$ and $m_0 < m_1 < m_2$. $P_i = 1/3$, Costs associated with the decisions are $C_{ii} = 0$, for $i = 0, 1, 2$ and $C_{ij} = 1$ when $I \neq j$. Find appropriate detection thresholds for this problem so that the probability of error is minimized? [12]

8.a) Derive the likelihood ratio test metrics used in discrete-time correlator and analog correlator.

b) Show the equivalence between correlator and matched filter receiver. Also show how matched filters can be used for hypothesis testing problems. [12]

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